EFFECT OF SURFACE TENSION AND VISCOSITY ON THE COLLAPSE OF A CAVITATION BUBBLE

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 5, pp. 903–908, 1968

UDC 532.1:532.528

The effect of surface tension and viscosity forces on the rate of collapse of a cavitation vapor bubble has been quantitatively estimated. The results of corresponding computations of the cavity collapse rate obtained by exact and approximate methods are compared.

The radial motion of a spherical vapor bubble in an incompressible fluid is described by the equation [1]

$$\beta\ddot{\beta} + \frac{3}{2}\dot{\beta}^2 + \frac{C}{\beta}\dot{\beta} + \frac{2D}{\beta} + 1 = 0. \qquad (1)$$

If we assume that the effect of the surface tension forces and viscosity can be neglected, we can easily evaluate the first integral of the differential equation (1) [2]:

$$\dot{\beta}^2 = \frac{2}{3\beta^3}(1-\beta^3). \qquad (2)$$

If we set C = 0 in Eq. (1), it can also be integrated

$$\dot{\beta}^2 = \frac{2}{3\beta^3}(1-\beta^3)\left[1 + 3D\frac{1-\beta^2}{1-\beta^3}\right]. \qquad (3)$$

The effect of the surface tension forces on the rate of collapse of a cavitation bubble will be small if

$$D < \frac{1}{3}. \qquad (4)$$

As a result of an analysis and numerical integration of Eq. (1), for the case D = 0 it has been shown [3–5] that there is a critical value of the initial radius R_0^* such that at $R_0 < R_0^*$ the cavity collapse time becomes infinitely large.

The viscosity effect is significant only at $\beta \ll 1$, when the velocity of the cavity boundary is sufficiently large. In this stage, relation (2) can be written approximately in the form

$$\dot{\beta}^2 = \frac{2}{3\beta^3}. \qquad (5)$$

We substitute into the viscosity term of Eq. (1) the value of the boundary velocity (5). In this case, the role of the viscous stresses is exaggerated, since the actual boundary velocity should be lower. Obviously, this applies only if condition (4) is satisfied. In this case, integration of Eq. (1) leads to the result

$$\dot{\beta}^2 = \frac{2}{3\beta^3}(1-\beta^3)\times$$
$$\times\left(1 + 3D\frac{1-\beta^2}{1-\beta^3} - 2\sqrt{6}C\frac{1-\beta^{1/2}}{1-\beta^3}\right). \qquad (6)$$

When $\beta < 1$, so that $\beta^3 \ll 1$, expression (6) can be simplified:

$$\dot{\beta}^2 = \frac{2}{3\beta^3}\left[1 + 3D(1-\beta^2) - 2\sqrt{6}C(1-\beta^{1/2})\right]. \qquad (7)$$

In the final stages of motion, when $\beta \ll 1$, relation (6) can be further simplified:

$$\dot{\beta}^2 = \frac{2}{3\beta^3}(1 + 3D - 2\sqrt{6}C). \qquad (8)$$

Using relations (6), (7), and (8), we can compute the velocity of the boundary of the collapsing cavity in various stages of motion.

When D = 0, relation (8) takes the value

$$\dot{\beta}^2 = \frac{2}{3\beta^3}(1 - 2\sqrt{6}C), \qquad (9)$$

from which it follows that the effect of viscosity on the collapse rate will be small if $C < (2\sqrt{6})^{-1}$. The velocity vanishes if $C = (2\sqrt{6})^{-1}$. In this case, the value of the initial radius should approximately correspond to the critical value R_0^*. With further increase in the parameter C, relation (9) becomes meaningless, since the velocity becomes an imaginary quantity.

For the collapse of a cavity in water under atmospheric pressure, the exact value of the critical initial radius $R_0^* = 8 \cdot 10^{-7}$ m [4], and that calculated from Eq. (9) is $R_0^* = 2 \cdot 10^{-6}$ m. The agreement is quite satisfactory, indirect evidence of the accuracy of the proposed solution. As was to be expected, owing to the exaggeration of the role of viscosity, the critical radius is likewise overestimated.

It follows from (8) that a situation may exist in which the investigated factors mutually compensate each other,

$$3D - 2\sqrt{6}C = 0 \qquad (10)$$

and the boundary velocity is given by the Rayleigh solution [2]: $K = \dot{\beta}\,[2/(3\beta^3)]^{-1/2} = 1$.

It follows from (10) that, irrespective of the dependence on the initial radius, the surface tension and viscosity forces mutually compensate each other if the external pressure has the value

$$p_0^* = 2.34 \cdot 10^{-2}\frac{\sigma^2\rho}{\mu^2}. \qquad (11)$$

The effect of the investigated factors will be small ($K \approx 1$) if

$$|3D - 2\sqrt{6}C| \ll 1. \qquad (12)$$

Table 1

The Error Δ%

R_0, m	p_0, kg · m^{-2}						
	10^7	10^6	10^5	$1.32 \cdot 10^4$	10^4	10^3	10^2
10^{-2}	$-3 \cdot 10^{-4}$	$-9 \cdot 10^{-4}$	$-2 \cdot 10^{-3}$	0	$1.4 \cdot 10^{-3}$	$8 \cdot 10^{-2}$	1
10^{-3}	$-3 \cdot 10^{-3}$	$-9 \cdot 10^{-3}$	$-2 \cdot 10^{-2}$	0	$1.4 \cdot 10^{-2}$	$8 \cdot 10^{-1}$	10
10^{-4}	$-3 \cdot 10^{-2}$	$-9 \cdot 10^{-2}$	$-2 \cdot 10^{-1}$	0	$1.4 \cdot 10^{-1}$	8	
10^{-5}	$-3 \cdot 10^{-1}$	$-9 \cdot 10^{-1}$	-2	0	1.4		

In this case, by means of a Taylor expansion expression (8) can be reduced to the form

$$K = \dot{\beta}\,[2/(3\beta^3)]^{-1/2} = 1 + 1.5D - \sqrt{6}\,C. \qquad (13)$$

It is convenient to refer the external pressure to its equilibrium value given by relation (11), $\varepsilon = p_0/p_0^*$; then $p_0 = 2.34 \cdot 10^{-2} \varepsilon \sigma^2 \rho \mu^{-2}$ and the expression for the error introduced by the action of the viscosity and surface tension forces into the value of the boundary velocity is given by the relation

$$\Delta\ \% = 6.4 \cdot 10^3 \mu^2 (\sigma \rho \varepsilon R_0)^{-1} (1 - \varepsilon^{1/2}). \qquad (14)$$

Interest is usually focused on cavitation in water ($\sigma = 7.5 \cdot 10^{-3}$ kg · m^{-1}, $\mu = 10^{-4}$ kg · sec · m^{-2}, $\rho = 10^2$ kg · sec^2 · m^{-4}); in this particular case,

$$\Delta\ \% = 8.5 \cdot 10^{-5} (1 - \varepsilon^{1/2}) (\varepsilon R_0)^{-1}. \qquad (15)$$

The results of calculations based on (11) and (15) are presented in Table 1.

The calculations show that the viscosity and surface tension effects mutually compensate each other under the conditions of cavity collapse most often encountered in practice at a pressure close to atmospheric. In these circumstances, irrespective of the initial radius the error is close to zero.

At pressures above atmospheric, viscosity predominates, the velocity given by formula (2) is too high, and the error quickly decreases with increase in pressure. At $R_0 = 10^{-5}$ m, the error is about 2% in the range $10^5 < p_0 < 2 \cdot 10^4$ kg · m^{-2}.

At pressures below atmospheric, surface tension predominates, the velocity calculated from (2) is too low, and the error increases rapidly as the pressure falls. However, even at R_0 = 1 mm and $p_0 = 10^2$ kg · m^{-2} the error is only 10%.

As a check we obtained a numerical solution of Eq. (1) on an M-20 computer for water at four values of R_0 (10^{-7}; $8 \cdot 10^{-7}$; $5 \cdot 10^{-6}$, and 10^{-5} m) and three values of p_0 (10^3, 10^4, and 10^5 kg · m^{-2}).

The calculations were made for three cases: neither viscosity nor surface tension; viscosity only; viscosity and surface tension combined.

The results for $R_0 \leq 8 \cdot 10^{-7}$ m were plotted in the form of graphs of the dimensionless radius β and velocity $\dot{\beta}$ of the cavity versus dimensionless time τ (Figs. 1 and 2). The calculations established that, for water, taking viscosity into account has little effect on the dynamic characteristics of the cavity, except for the unimportant case $R_0 \leq 8 \cdot 10^{-7}$ m, which corresponds to $C > 0.5$. The total collapse time increases

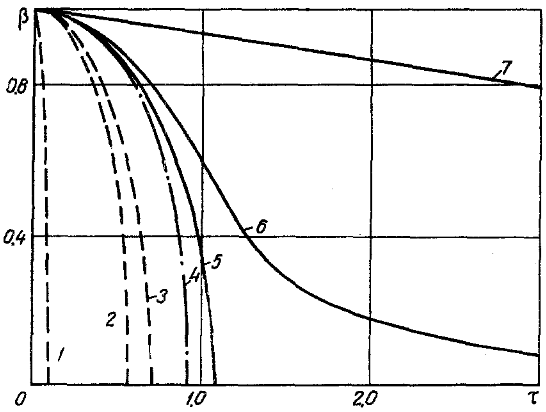

Fig. 1. Radius of a spherical bubble as a function of time: 1–3) real liquid ($C \neq 0$, $D \neq 0$); 4) ideal liquid ($C = 0$, $D = 0$); 5–7) viscous liquid ($C \neq 0$, $D = 0$): 1, 7) $p_0 = 10^3$ kg/m^2, $R_0 = 10^{-7}$ m; 2, 6) $p_0 = 10^5$ kg/m^2, $R_0 = 10^{-7}$ m; 3, 5) $p_0 = 10^4$ kg/m^2, $R_0 = 8 \cdot 10^{-7}$ m.

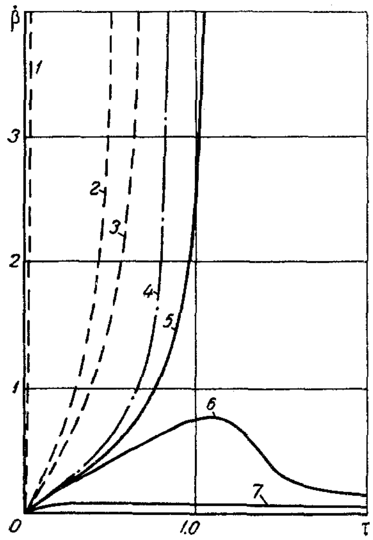

Fig. 2. Rate of collapse of a spherical bubble as a function of time. 1–3) real liquid ($C \neq 0$, $D \neq 0$); 4) ideal liquid ($C = 0$, $D = 0$); 5–7) viscous liquid ($C \neq 0$, $D = 0$): 1, 7) $p_0 = 10^3$ kg/m^2, $R_0 = 10^{-7}$ m; 2, 6) $p_0 = 10^5$ kg/m^2, $R_0 = 10^{-7}$ m; 3, 5) $p_0 = 10^4$ kg/m^2, $R_0 = 8 \cdot 10^{-7}$ m.

Table 2

Comparison of Results of Approximate and Exact Solutions

p_0, kg/m²	β	R_0, m							
		10^{-7}		$8\cdot10^{-7}$		$5\cdot10^{-6}$		10^{-5}	
		K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
10^3	0.8	12.4	10.3	4.48	4.26	2.01	1.98	1.59	1.58
	0.6	13.0	8.4	4.70	4.27	2.09	2.04	1.64	1.63
	0.2	13.6	3.2	4.90	3.55	2.16	2.01	1.68	1.635
	0.1	13.6	0.86	4.90	3.30	2.16	1.86	1.68	1.62
10^4	0.8	3.64	3.15	1.59	1.63	1.12	1.135	1.06	1.065
	0.6	3.72	2.78	1.62	1.58	1.12	1.135	1.06	1.07
	0.4	3.68	2.12	1.60	1.52	1.12	1.110	1.06	1.06
	0.2	3.45	1.04	1.54	1.38	1.11	1.106	1.05	1.05
	0.1	3.18	0.278	1.46	1.26	1.09	1.080	1.05	1.05
10^5	0.8	1.13	1.315	1.01	1.05	1	1.03	1	1
	0.6	1.05	1.160	1.00	1.02	1	1.01	1	1
	0.4	0.77	0.908	0.975	1.00	1	1	1	1
	0.2	Im	0.525	0.915	0.948	0.99	0.997	1	1
	0.1	Im	0.215	0.87	0.91	0.98	0.987	0.99	1

slightly. At $R_0 = 8 \cdot 10^{-7}$ m, the collapse time becomes infinite, and the velocity infinitely small, which is consistent with the results of [3–5].

When the surface tension forces are taken into account, the total collapse time remains finite and less than that calculated by Rayleigh [2] at all the investigated values of the dimensionless viscosity parameter.

For cavities with an initial radius $R_0 \geq 5 \cdot 10^{-6}$m at an above-atmospheric external pressure, the viscosity and surface tension effects mutually compensate each other.

Thus, qualitatively these conclusions correspond to those that follow from the approximate solution of the problem presented above.

The results of a comparison of the parameter $K = \dot{\beta}\,[2(1-\beta^3)/(3\beta^3)]^{-1/2}$, calculated approximately (K_1) and exactly (K_2) are presented in Table 2. As was to be expected, the agreement is satisfactory when the dimensionless viscosity parameter C is less than 0.2.

NOTATION

$\beta = R/R_0$ is the dimensionless cavity radius; R is the variable radius; R_0 is the initial radius; $\dot{\beta} = d\beta/d\tau$ is the dimensionless velocity; $\ddot{\beta} = d^2\beta/d\tau^2$ is the dimensionless acceleration; $\tau = t/R(p_0/\rho)^{1/2}$ is the dimensionless time; t is the time; ρ is the density of the liquid; p_0 is the difference between the pressure at infinity and the saturated vapor pressure; $C = 4\mu/R_0(p_0\rho)^{1/2}$ is the dimensionless viscosity parameter; $D = \sigma/R_0p_0$ is the dimensionless surface tension parameter; μ is the dynamic viscosity; σ is the surface tension.

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29 May 1967